Tabulation of knots in $T \times I$

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Motivation

1. T x I is one of the simplest 3-manifolds after the 3-sphere.
2. Knots in T x I can be presented by diagrams.
3. Two diagrams represent isotopic knots iff they are related by Reidemeister moves.
4. Tables of classical knots are useful.
Definition

• Two knots in $T \times I$ are equivalent if there is a homeomorphism $T \times I \rightarrow T \times I$ taking one knot onto the other.
Definition

• A knot $K$ in $T \times I$ is not prime if at least one of the following holds:
  
  1. $K$ is local (contained in a ball in $T \times I$)
  2. $K$ crosses a ball along a knotted arc.
  4. $K$ admits a diagram composed of two diagrams having geometric degree one.
Enumeration of knots

• 1. Enumerate regular graphs.

![Diagram of knots and graphs]
Why no loops?

- If there is a trivial loop, then the diagram is not minimal.

- If there is a nontrivial loop, then the knot is contained in an annulus.
Then for each graph we enumerate all corresponding projections

- **Hint:** a valence 4 graph is a projection if

- The “straight forward” rule determines a connected closed curve.
Why no duplicates?

• Answer: Generalized Kauffman polynomials of all knots with 5 or less crossings turned to be different!

\[ X(K) = (-a)^{-3w(K)} \sum_{s} \alpha(s) - \beta(s) (-a^2 - a^{-2})^\gamma(s) x \delta(s) \]

• where \( \alpha(s) \) and \( \beta(s) \) are the numbers of markers A and B in a given state \( s \), and
• \( \gamma(s) \), \( \delta(s) \) are the numbers of trivial and nontrivial circles in \( T \) obtained by resolving all crossing points. Just as for the original Kauffman polynomial, the sum is taken over all states. Of course, \( w(K) \) is the writhe of the diagram. Direct manual and computer calculations showed that all Kauffman polynomials are of the above knots are different, See Table 1. Therefore the corresponding knots are also are different.
Genus one prime virtual knots
THE END