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Symmetric unions indistinguishable by knot Floer and Khovanov homology

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Knot homology

$$\Longrightarrow \quad H(C_{i,j}, \partial)$$

◊ Knot Floer homology $\widehat{HFK}(K)$

(P. Ozsváth-Z. Szabó 2004, J. Rasmussen, 2002)

◊ Khovanov homology $Kh(K)$

(M. Khovanov, 2000)

Classical knot invariants

⋆ Alexander polynomial
⋆ Jones polynomial
⋆ Determinant
⋆ Signature
⋆ etc.

Knot homologies are invariants which are Powerful and Computable.

Euler Characteristics for Knot homologies

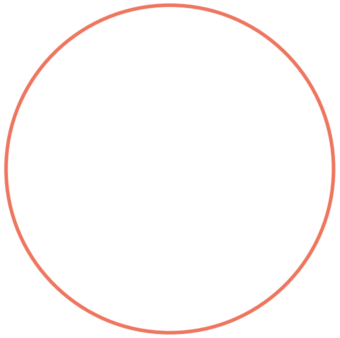
⋆ The Euler Characteristic of Knot Floer homology is the Alexander polynomial.

⋆ The Euler Characteristic of Khovanov homology is the Jones polynomial.

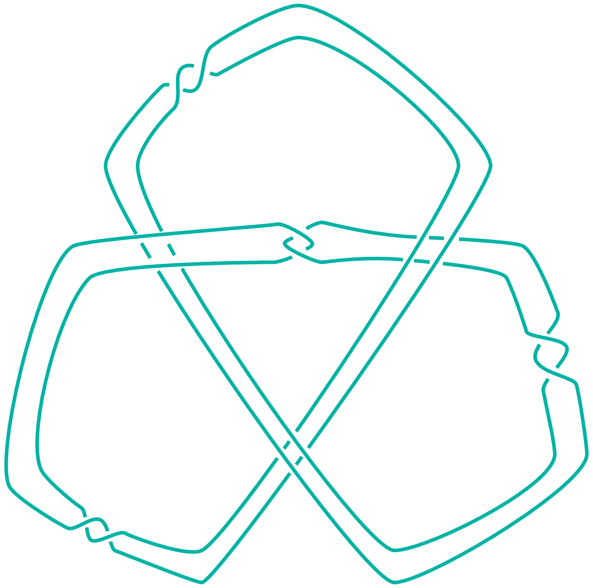
Fact

There exists a pair of knots
with the same Alexander module,
but different Knovanov homology
and different knot Floer homology.

Example.



K



$\hat{K}$

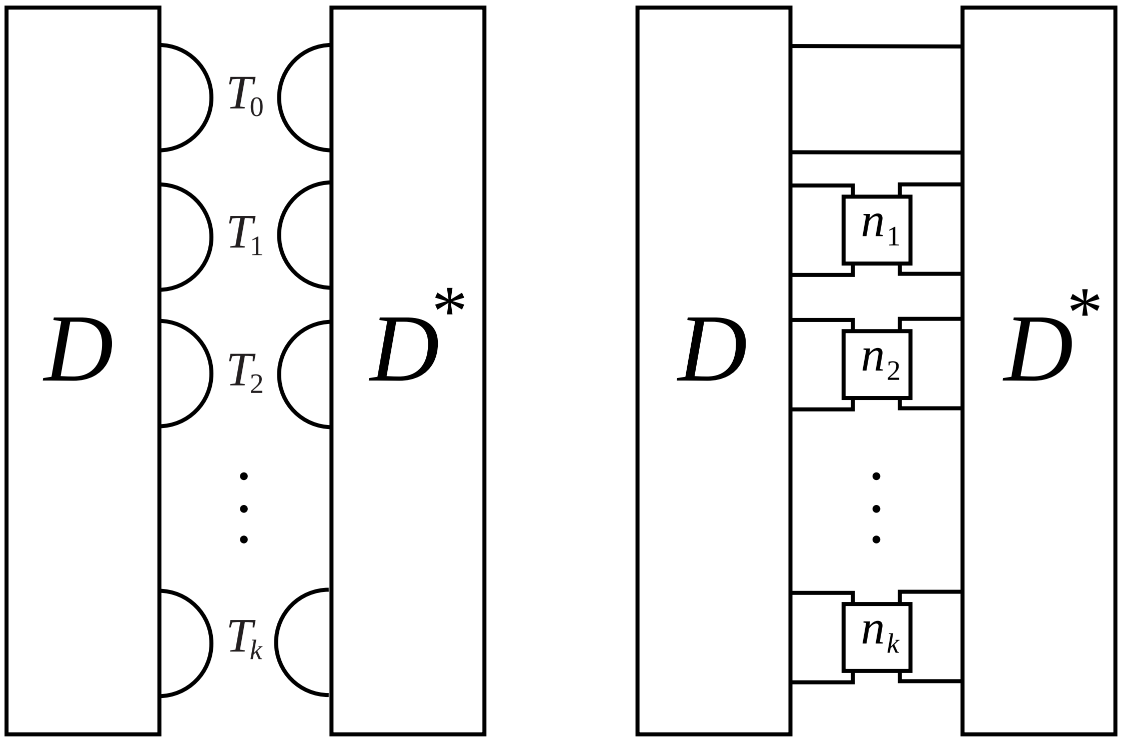
$Kh(K) \neq Kh(\hat{K})$ (M.Hedden-L. Watson, 2008),

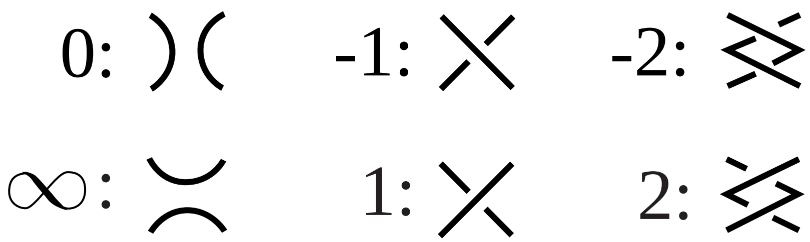
$\widehat{HFK}(K) \neq \widehat{HFK}(\hat{K})$ (P. Ozsváth-Z. Szabó, 2004).

Question.

Is there a pair of knots which have the same Khovanov homology and the same knot Floer homology, but different Alexander modules?

Symmetric unions





$D \cup D^*(n_1, \ldots, n_k)$

Fact

$\star$ Every symmetric union is a ribbon knot.

$\star$ Δ: the Alexander polynomial.

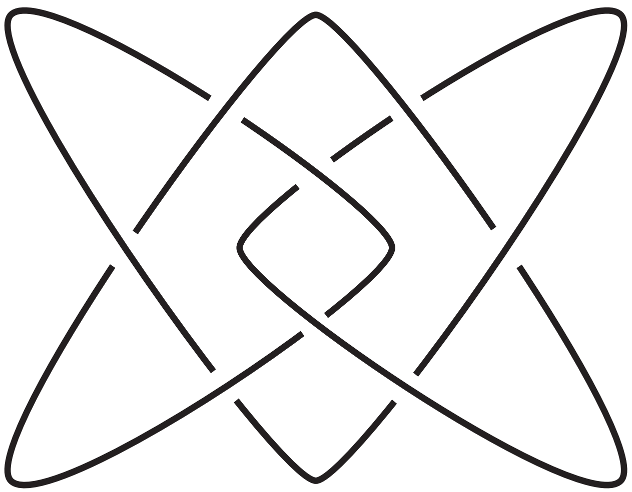
$\Delta(D \cup D^*(n_1, \ldots, n_k)) = \Delta(D \cup D^*(n'_1, \ldots, n'_k))$ if $n_i \equiv n'_i$ (mod 2) for all i.

$\star$ $det(D \cup D^*(n_1, \ldots, n_k)) = det(D)^2$.

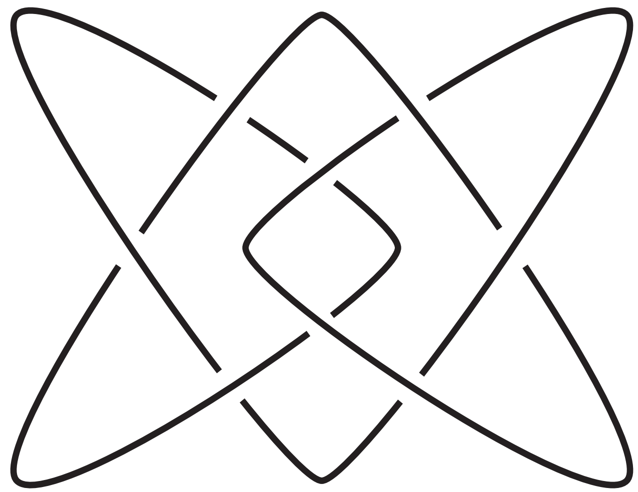
(C. Lamm (2000))

Theorem 1.

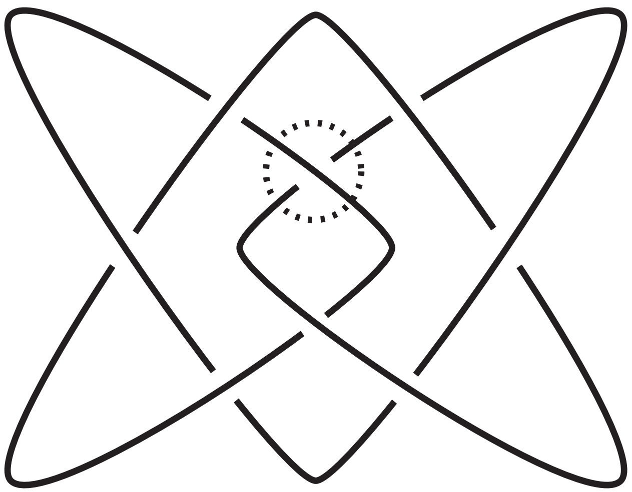
The knot 8_{20} and the connected sum of a trefoil knot and its mirror image have the same knot Floer homology, but different Alexander modules.



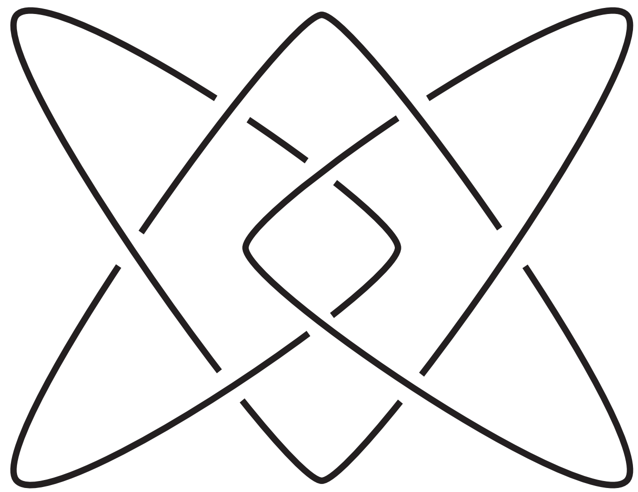
8_{20}



$3_1 \# 3_1^*$



8_{20}



$3_1 \# 3_1^*$

Theorem 2.

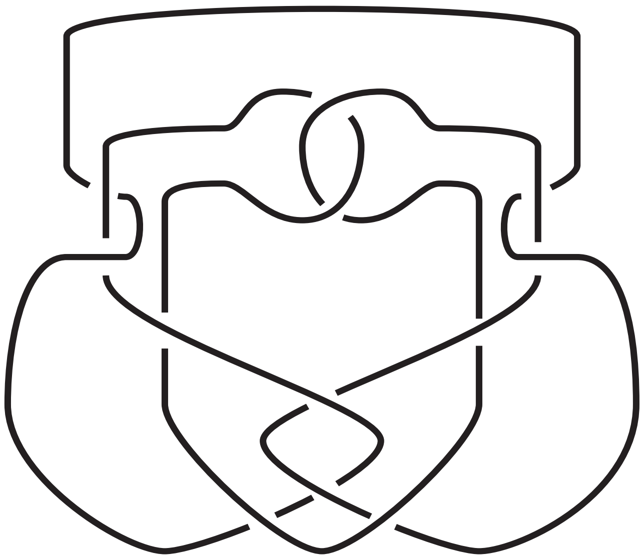
There exist a pair of symmetric unions of two-bridge knots with the same Knovanov homology and knot Floer homology, but different Alexander modules.

Proof. We use skein exact sequence.

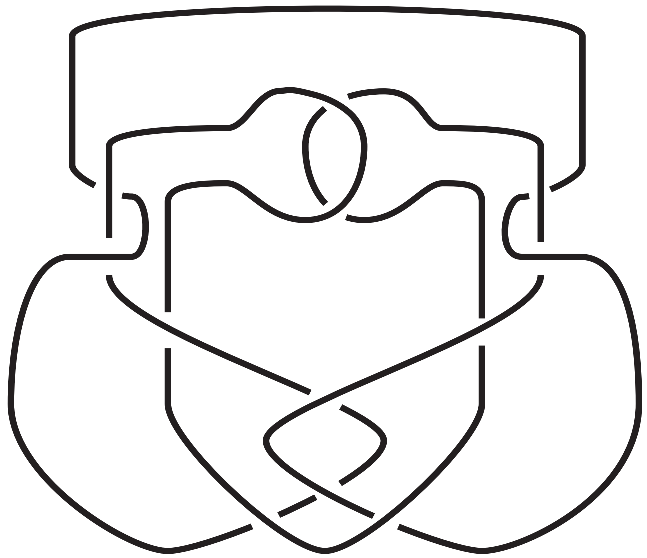
For some pair of knots K and $\hat{K}$,

$Kh(K) = Kh(\hat{K})$ (L. Watson, Algebr. Geom. Top. 2007),

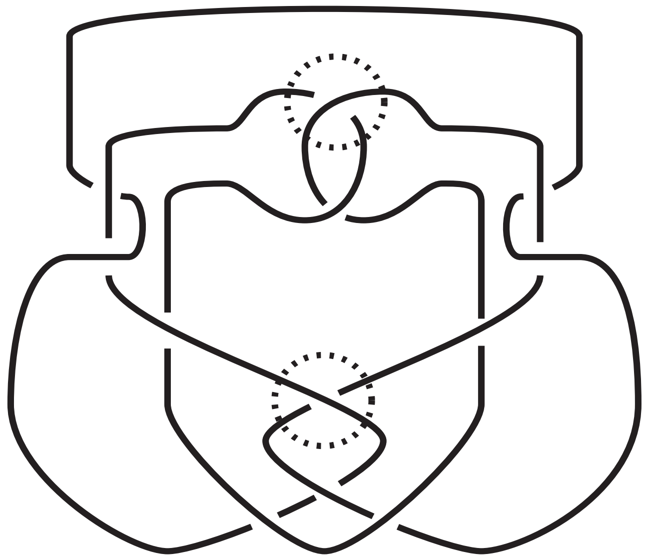
$\widehat{HFK}(K) = \widehat{HFK}(\hat{K})$ (Ozsváth-Szabó, Top. Appl. 2004).



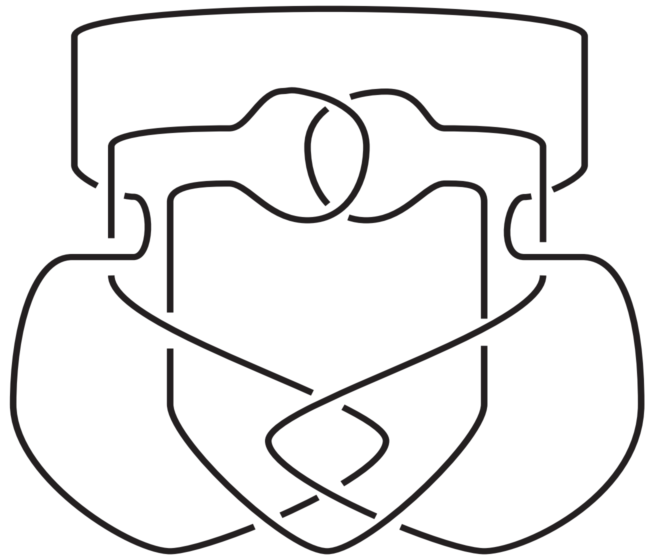
$\hat{K}$



$4_1 \# 4_1$



$\hat{K}$



$4_1 \# 4_1$

Fact

Fact. (L. Watson (2007))

There exists a pair of prime knots with identical Khovanov homology but distinct HOMFLYPT polynomials.

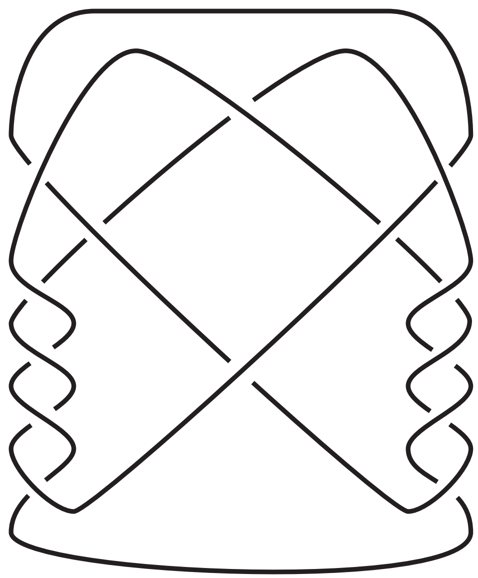
Theorem. (L. Watson (2007))

There exists an infinite family of distinct knots with identical Khovanov homology.

Theorem 3.

There exists a pair of symmetric unions of two-bridge knots which have the same Khovanov homology, the same knot Floer homology and the same HOMFLYPT polynomial, but different Kauffman polynomials.

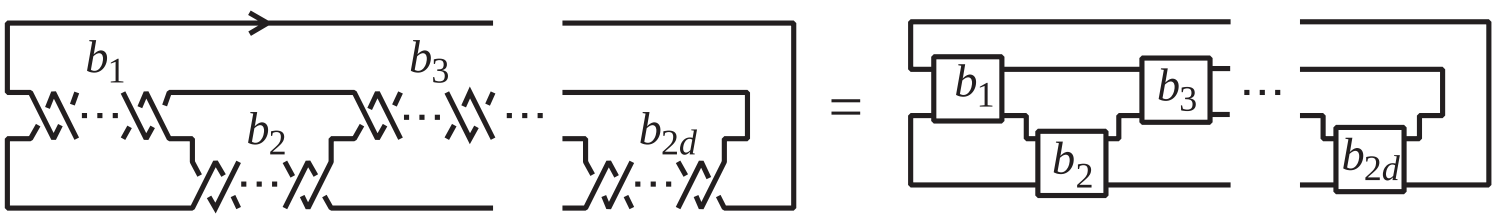
Example. (10_{48}, 10^*_{48})



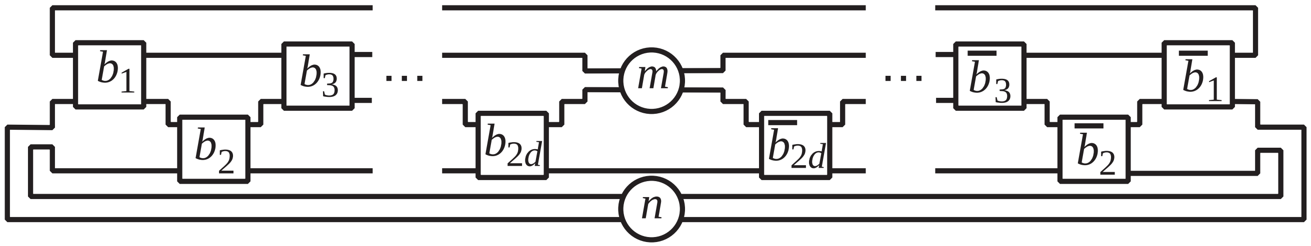
10_{48}

A symmetric union of a two-bridge knot

$T(b_1, \ldots, b_{2d}) =$



$B(m, n)(T(b_1, \ldots, b_{2d})) =$



Lemma 4.

For any integer ℓ, we have the following.

(1) $Kh(B(m,n)(T(b_1,\ldots,b_{2d}))) =$

$Kh(B(m+\ell,n-\ell)(T(b_1,\ldots,b_{2d})))$,

(2) $\widehat{HFK}(B(1,0)(T(b_1,b_2,\ldots,b_{2d}))) =$

$\widehat{HFK}(B(-1,0)(T(b_1,b_2,\ldots,b_{2d})))$.

Proposition 5.

If $b_i = b_{2d-i+1} (1 \leq i \leq d)$, we have the following.

(1) $Kh(B(1,0)(T(b_1, b_2, \ldots, b_{2d}))) =$

$Kh(B(0,-1)(T(b_1, b_2, \ldots, b_{2d})))$,

(2) $\widehat{HFK}(B(1,0)(T(b_1, b_2, \ldots, b_{2d}))) =$

$\widehat{HFK}(B(0,-1)(T(b_1, b_2, \ldots, b_{2d})))$.

Corollary 6.

There exists an infinite family of pairs of symmetric unions which have the same Khovanov homology and the same knot Floer homology, but different HOMFLYPT polynomials.

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Thank you