A New Obstruction of Quasi-alternating Links

Khaled Qazaqzeh
joint work with Nafaa Chbili

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The Main Idea

For any quasi-alternating link $L$, we have $\deg Q_L \leq \det(L) - 1$. Khaled Qazaqzeh joint work with Nafaa Chbili

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3. Results of the main theorem.
4. Questions and Conjectures
Quasi-alternating links were first introduced by Ozsváth and Szabó in 2006. This class of links arises as a natural generalization of alternating links. Quasi-alternating links are defined recursively as follows:

**Definition**

The set $Q$ of quasi-alternating links is the smallest set satisfying the following properties:

1. The unknot belongs to $Q$.
2. If $L$ is a link with a diagram $D$ containing a crossing $c$ such that both smoothings of the diagram $D$ at the crossing $c$, $L_0$ and $L_\infty$, as in the figure below belong to $Q$, then $L$ is in $Q$.
3. If both smoothings of the diagram $D$ at the crossing $c$, $L_0$ and $L_\infty$, belong to $Q$, then $\det(L) = \det(L_0) + \det(L_\infty)$ holds.

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  - Both smoothings of the diagram $D$ at the crossing $c$, $L_0$ and $L_\infty$, as in the figure below belong to $Q$,
  - $\det(L_0)$, $\det(L_\infty) \geq 1$,
  - $\det(L) = \det(L_0) + \det(L_\infty)$;

then $L$ is in $Q$ and in this case we say that $L$ is quasi-alternating at the crossing $c$ with quasi-alternating diagram $D$.
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Figure: The diagram of the link $L$ at the crossing $c$ and its smoothings $L_0$ and $L_\infty$ respectively.
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3. the $\mathbb{Z}/2\mathbb{Z}$ knot Floer homology of any quasi-alternating link is thin [Manolescu and Ozsváth];
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the reduced odd Khovanov homology of any quasi-alternating link is thin [Ozsváth, Rasmussen and Szabó].
The Brandt, Lickorish and Millet link polynomial invariant $Q_L(x)$ is a Laurent polynomial which is defined as follows:

$$Q_L(x) = 1;$$

$$Q_L^+ + Q_L^- = x(Q_L^0 + Q_L^∞).$$

where $L^+$, $L^-$, $L^0$ and $L^∞$ are four link diagrams which are identical except in a small ball where they are as in the following figure:
The Brandt, Lickorish and Millet link polynomial invariant $Q_L(x)$ is a Laurent polynomial which is defined as follows:

1. $Q_U(x) = 1$;

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Figure: $L_+$, $L_-$, $L_0$ and $L_\infty$, respectively.
The Proof of the Main Theorem

Theorem

For any quasi-alternating link $L$, we have $\deg Q_L \leq \det(L) - 1$. 

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A New Obstruction of Quasi-alternating Links
The Proof of the Main Theorem

**Theorem**

For any quasi-alternating link $L$, we have $\deg Q_L \leq \det(L) - 1$.

**Lemma**

Let $L$ be a link, then

$$\deg Q_L \leq \max\{\deg Q_{L_0}, \deg Q_{L_\infty}\} + 1,$$

where $L_0, L_\infty$ are the smoothings of the link $L$ at any crossing $c$. 
Proof.

We use induction on the determinant of the given quasi-alternating link $L$. The result is obvious if $\det(L) = 1$. If $L$ is a quasi-alternating link with determinant $m + 1$, then both $\det(L_0)$ and $\det(L_\infty)$ are less than or equal to $m$. By the induction assumption $\deg Q_{L_0} < \det(L_0)$ and $\deg Q_{L_\infty} < \det(L_\infty)$. Consequently:

$$\deg Q_L \leq \max\{\deg Q_{L_0}, \deg Q_{L_\infty}\} + 1$$
$$< \max\{\det(L_0), \det(L_\infty)\} + 1$$
$$< \det(L_0) + \det(L_\infty) = \det(L).$$
Corollary

There are only finitely many Kanenobu knots that are quasi-alternating.
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Figure: The diagram of the Kanenobu knot $K(p, q)$ with $p, q > 0$
Proof.

A necessary condition for the Kanenobu knot to be quasi-alternating, is as follows:

\[ \deg Q_{K(p,q)} \leq |p| + |q| + 6 < 25. \]

This implies that \(|p| + |q| < 19\) and we know that there are only finitely many values of \(p\) and \(q\) that satisfy this inequality. \(\square\)
Corollary

The Montesinos link
$L = M(e; (\alpha_1, \beta_1), (\alpha_2, \beta_2), \ldots, (\alpha_{r-1}, \beta_{r-1}), (\alpha, \beta))$ for all
$\alpha = l + k\beta$ with $k$ large enough and $l = 0, 1, \ldots, \beta - 1$ in standard
form is not quasi-alternating if $e = 1$ and $\sum_{i=1}^{r-1} \frac{\beta_i}{\alpha_i} = 1$ and this
supports the following conjecture that characterizes
quasi-alternating Montesinos links:
Figure: The standard diagram of Montesinos link
Conjecture (Q,C,Qublan)

The Montesinos link $L = M(e ; (\alpha_1, \beta_1), (\alpha_2, \beta_2), \ldots, (\alpha_r, \beta_r))$ in standard form is quasi-alternating iff one of the following conditions is satisfied

1. $e \leq 0$;
2. $e \geq r$;
3. $e = 1$ with $\frac{\alpha_i}{\alpha_i - \beta_i} > \min\{\frac{\alpha_j}{\beta_j} \mid j \neq i\}$ for some $1 \leq i \leq r$;
4. $e = r - 1$ with $\frac{\alpha_i}{\beta_i} > \min\{\frac{\alpha_j}{\alpha_j - \beta_j} \mid j \neq i\}$ for some $1 \leq i \leq r$.

Remark

We proved one direction of the above conjecture and our corollary proves part of the other direction of the conjecture.
A necessary condition for the above Montesinos link to be quasi-alternating is as follows:

\[ \text{deg } Q_L = c(D) - 2 = c(L) - 2 < \det(L) = \beta \prod_{i=1}^{r} \alpha_i, \]

Note that increasing the value of \( k \) will increase the value of \( c(D) \) while the determinant stays fixed. Therefore, we can choose \( k \) large enough for fixed \( \beta \) so that \( \beta \prod_{i=1}^{r} \alpha_i \leq c(L) - 2 \).

For the second claim, we show that

\[ \frac{\alpha_i}{\alpha_i - \beta_i} \leq \min\{\min\{\frac{\alpha_j}{\beta_j} \mid j \neq i\}, \frac{\alpha}{\beta}\} \text{ for any } 1 \leq i \leq r \text{ and } \frac{\alpha}{\alpha - \beta} \leq \min\{\frac{\alpha_j}{\beta_j} \mid 1 \leq j \leq r\} \text{ by contradiction for } k \text{ large enough.} \]
Remark

The above Corollary detects that most but finitely many of the Montesinos links of the form
\[ L(m, n) = M(0; (m^2 + 1, m), (n, 1), (m^2 + 1, m)) = M(1; (m^2 + 1, m), (n, 1), (m^2 + 1, m - m^2 - 1)) \]
for positive integers \( m, n \) and large \( n \) for fixed \( m \) are not quasi-alternating.

This phenomena was first detected by Greene for the Montesinos knot \( L(2, 3) \) which is the knot 11n50. Also, Greene pointed out that this can be generalized easily to all \( L(m, n) \) with \( m > n \).

Remark

The inequality in the main theorem does not characterize quasi-alternating links since the knots 9_46, 10_128, and 11n50 for instance, satisfy the inequality \( \deg Q_L < \det(L) \), but they are not quasi-alternating.
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The inequality in the main theorem does not characterize quasi-alternating links since the knots $9_{46}, 10_{128},$ and $11n50$ for instance, satisfy the inequality $\deg Q_L < \det(L)$, but they are not quasi-alternating.
Proposition

There are two infinite families one of knots and one of links consisting of links that are not quasi-alternating but satisfy the inequality in the main theorem.

Proof.

The first family is the set of the pretzel knots of the form $P(r+2, r+1, -r)$ and the second family is the set of the pretzel links $P(r+1, r+1, -r)$, where $r > 3$ is an odd integer. We show that these knots and links are thick in Khovanov homology. Therefore, they are not quasi-alternating. However, they satisfy the inequality in the main theorem.

$$\deg Q P(r+2, r+1, -r) = 3r+1 \leq r^2 - 2 = \det(P(r+2, r+1, -r))$$

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There is an infinite family of links consisting of links that are not quasi-alternating homologically thin in Khovanov homology and satisfy the inequality in the main theorem.

Proof.
The family is the set of the pretzel links $P(n,n,-n)$ for $n \geq 3$. We show that all these links are homologically thin in Khovanov homology. However, Greene shows that they are not quasi-alternating. It is left to show that all these links satisfy the inequality in the main Theorem. We have

$$\deg Q_{P(n,n,-n)} = 3n - 2 \leq n^2 = \det(P(n,n,-n))$$

for $n \geq 3$. 

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A New Obstruction of Quasi-alternating Links
Questions and Conjectures

Question

*Can we determine all Kanenobu knots that are quasi-alternating?*

We think that $K(0,0) = 4$, $K(1,0) = 8$, $K(1, -1) = 8$, $K(2, -1) = 10$, $K(2, 0) = 10$, $K(1, 1) = 10$ are the only Kanenobu knots that are quasi-alternating.

Question

Can we characterize all quasi-alternating knots with crossing number less than or equal to 11? The table below combined with the table in given by Jablan gives a partial solution for the above question.
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We think that $K(0, 0) = 4_1 \# 4_1$, $K(1, 0) = 8_8$, $K(1, -1) = 8_9$, $K(2, -1) = 10_{129}$, $K(2, 0) = 10_{137}$, $K(1, 1) = 10_{155}$ are the only Kanenobu knots that are quasi-alternating.
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**Table:** Knot table

Khaled Qazaqzeh joint work with Nafaa Chbili

A New Obstruction of Quasi-alternating Links
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</table>

**Table:** Link table
Remark

We would like to mention that a refinement of our inequality was obtained by Teragaito in which he gives a sharper inequality as follows:

Theorem (Teragaito)

For any quasi-alternating link $L$. If $L$ is not a $(2, n)$-torus link, then $\deg Q_L \leq \det(L) - 2$. 
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Conjecture (Q, Qublan, and Jaradat)

For any quasi-alternating link $L$, we have

$$c(L) \leq \det(L).$$
Conjecture (Q, Qublan, and Jaradat)

For any quasi-alternating link $L$, we have

$$c(L) \leq \det(L).$$

Conjecture (Greene)

There exist only finitely many quasi-alternating links with a given determinant.
Thank you